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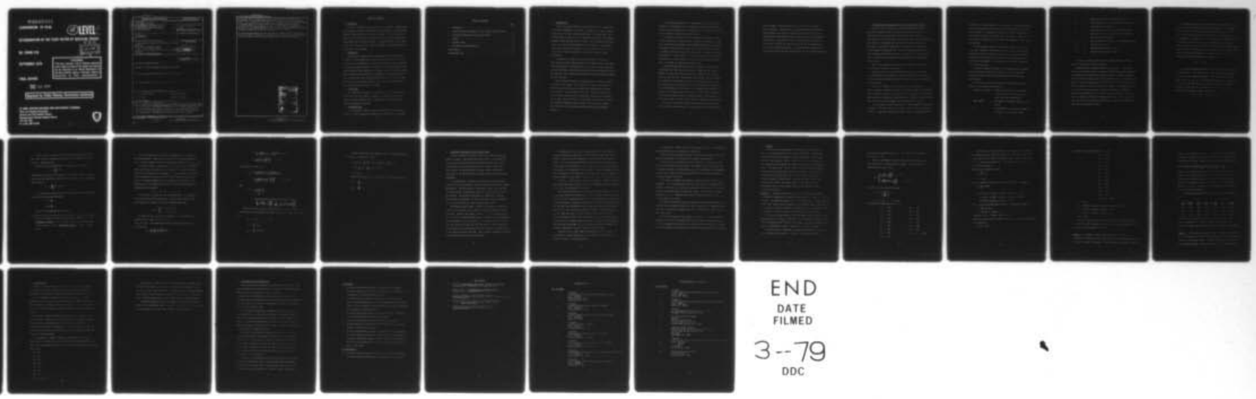

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DETERMINATION OF THE FLOAT FACTOR BY QUEUEING THEORY

DR. FRANK FOX

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FINAL REPORT

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US ARMY AVIATION RESEARCH AND DEVELOPMENT COMMAND
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One of the important problems which is of recurrent concern within the Department of Army is the accurate determination of the operational readiness float factor for a fleet of aircraft. If the float is too small, there will be a loss of service of those aircraft needing float replacements when none are available. On the other hand, if the float is too large, there will be a financial burden of providing float aircraft which are not needed.

The important question of the optimum size of the float can be answered by → next page

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recognizing that the demand for and utilization of float aircraft can be interpreted as a queueing system which can be analyzed by classical queueing theory. The solution of the queueing model provides statistics which represent the average number of aircraft needing float replacements when none are available and the average number of float aircraft which are available but unneeded. By evaluating these statistics for the various possible float sizes, the decision maker can select the optimum size of the float.

This methodology provides an excellent means of determining the ideal float size for a fleet of aircraft. It requires very little input data which should be easy to obtain, and it allows easy and complete sensitivity analysis.

EXECUTIVE SUMMARY

1. DISCUSSION:

One of the important problems which is of recurrent concern within the Department of Army is the accurate determination of the operational readiness float factor for a fleet of aircraft. If the float is too small, there will be a loss of service of those aircraft needing float replacements when none are available. On the other hand, if the float is too large, there will be a financial burden of providing float aircraft which are not needed.

2. METHODOLOGY:

The important question of the optimum size of the float can be answered by recognizing that the demand for and utilization of float aircraft can be interpreted as a queueing system which can be analyzed by classical queueing theory. The solution of the queueing model provides statistics which represent the average number of aircraft needing float replacements when none are available and the average number of float aircraft which are available but unneeded. By evaluating these statistics for the various possible float sizes, the decision maker can select the optimum size of the float.

3. CONCLUSIONS:

This methodology provides an excellent means of determining the ideal float size for a fleet of aircraft. It requires very little input data which should be easy to obtain, and it allows easy and complete sensitivity analysis.

4. RECOMMENDATIONS:

This methodology should be used to evaluate the consequences of floats of various sizes when determining the ideal size of the float.

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1. INTRODUCTION

A fleet of aircraft usually has associated with it a small group of aircraft designated as its float. According to AR 750-1, when an aircraft from the fleet requires maintenance for longer than a specified length of time, it is to be replaced in the fleet by an operationally ready aircraft from its float. The replacement which takes place is in the use and designation of the aircraft. The aircraft which needed maintenance becomes the float aircraft until it is repaired and is itself exchanged for an aircraft needing a float replacement.

The question of interest is how large should the float be? That question is made more difficult by the random nature of the needs for float aircraft. No matter what the size of the float, there will be times when float aircraft are needed and none are available; and for the same size float, there will be times when float aircraft are available but unneeded.

However those situations are exacerbated if the float is either too small or too large. If the float is too small, there will be a continual problem of aircraft needing float replacements when none are available. In this situation the fleet looses the services of those aircraft which need float replacement when none are available. On the other hand, if the float is too large, there will be a continual problem of available float aircraft when they are not needed. Thus there will be the financial burden of providing unnecessary aircraft.

The important question of the optimum size of the float can be answered by recognizing that the demands for float aircraft and the filling of those demands can be interpreted as a queueing system which may be analyzed by classical queueing theory.

In a queueing system, there are customers arriving for service of some kind and one or more servers to provide that service. In our case, the potential customers of the queueing system are the aircraft. The number of servers in the system is the number of float aircraft. A customer arrives for service when an aircraft is determined to need replacement by a float aircraft. A service begins when an aircraft is exchanged for a float aircraft, and that service ends when that aircraft is restored to operationally ready status.

A queue develops when there are more aircraft in need of float replacement than there are float aircraft. When there are fewer demands for float aircraft than there are available float aircraft, there are idle servers. The queueing model provides vital statistics which describe the important features of the queueing system. The most important of those statistics are the expected queue length and the expected number of idle servers. The expected queue length is a measure of the average number of aircraft needing float replacements when none are available, and the expected number of idle servers is the average number of float aircraft which are available but unneeded.

In order to choose the optimum size of the float, the decision maker must first choose weights for the expected queue length and the

expected number of idle servers; and, of course, they could be weighted equally. Since when one of these quantities decreases the other increases, the objective function is then chosen to be the weighted sum of the expected queue length and the expected number of idle servers. Since there are only a finite number of choices for the size of the float, the objective function can be evaluated for each of the possible float sizes. The optimum size of the float is then the one which gives the smallest value for the objective function.

2. A FINITE CALLING POPULATION, MULTIPLE SERVER QUEUEING MODEL

There are several different alternative mathematical models for describing queueing systems. Mathematical results describing the characteristics of the queueing system are available for many of these models. We need to select the elementary model which corresponds most closely to the demand and utilization of float aircraft. One manageable system which corresponds very closely to our situation is the queueing system which has a finite calling population (potential customers) and multiple servers. Of course the usual conditions apply, first come, first served, Poisson arrivals, and exponential service times.

First, the queueing system will be described in enough detail to facilitate the use of its results.

Queueing theory is the mathematical treatment of a situation in which customers seek service of some kind from one or more servers. If there are more customers than can be served, a waiting line develops for the service.

Service is provided on a first come, first served basis. When a customer is served, he leaves the queueing system and if present, a new customer is served. The arrival of the customers is not uniform, and there may be times when a waiting line has developed and other times when there are too few customers and the servers are not busy.

In our queueing system there are only a finite number of potential customers. Therefore our system is said to have a finite calling population. Assume that customers are generated for the queueing system

according to a Poisson process, ie. the number of customers generated until any specific time has a Poisson distribution (This is called a Poisson Input). An equivalent assumption is that the time between consecutive arrivals has an exponential distribution. Under this distribution, customers arrive at random but according to some fixed average rate.

In this queueing system, more than one customer can be served at a time. Therefore, our system is said to have multiple servers. Assume that there is exponential service time, ie. the probability distribution of the time until the next service completion has an exponential distribution. In other words, the service completions are a Poisson process.

In order to obtain a solution to this queueing system, assume that the system has reached a steady state, ie. that transient conditions (such as when the queueing system first starts) no longer exist.

We make use of the following notation and terminology which applies to the steady state condition.

state of the system = number of customers in the queueing system (includes both those in the queue and those being served)

queue length = the number of customers waiting to be served

P_n = the probability that there are exactly n customers in the queueing system

S	=	number of servers in the queueing system
λ_n	=	mean arrival rate of new customers when there are n customers in the system
μ_n	=	mean service rate for the overall system when n customers are in the system. (ie. the combined rate for all busy servers)
L	=	expected number of customers in the queueing system
L_q	=	expected queue length
W	=	expected waiting time in system
W_q	=	expected waiting time in queue

In order to obtain equations for the queueing model, the first step is to develop the balance equations.

Suppose that the queueing system is in any state n. The system can leave that state either by losing one customer and being in state n-1 or by gaining one customer and being in state n+1. However, it cannot leave the state n again unless it first reenters state n. Therefore, the number of times the system leaves the n^{th} state differs at most by one from the number of times it enters the n^{th} state. To determine the rate at which the system enters and leaves the n^{th} state, the number of entrances and departures in a period of time are divided by that time. Since the number of entrances and departures differ at most by one, the rates at which the system enters and leaves a state are essentially the same over a long period of time.

By equating the rate of entry and the rate of departure for each state, a balance equation is obtained for each state. For example for the 0^{th} state, the only way of entering the 0^{th} state is to be in state 1 and loose one customer. Thus the rate of entering state 0 is the probability of being in state 1, times the rate of departures when the system is in state 1, ie. $\mu_1 P_1$. Similarly the only way of leaving the 0^{th} state is to be in the 0^{th} state and to gain one customer. Thus, the rate of leaving state 0 is the probability of being in state 0 times the rate of arrival when the system is in state 0, ie. $\lambda_o P_o$. Therefore, the balance equation for state 0 is

$$\mu_1 P_1 = \lambda_o P_o .$$

To obtain the balance equation for state 1, note that the system can enter state 1 in two ways; be in state 0 and gain one customer or be in state 2 and loose one customer. Hence the rate of entering state 1 is $\lambda_o P_o + \mu_2 P_2$. The system can leave state 1 in two ways; be in state 1 and gain one customer or be in state 1 and loose one customer. Hence, the rate of leaving state 1 is $\lambda_1 P_1 + \mu_1 P_1$. Therefore, the balance equation for state 1 is

$$\lambda_o P_o + \mu_2 P_2 = \lambda_1 P_1 + \mu_1 P_1 .$$

Similarly the balance equations for the other states can be developed. The results are shown in the following table.

BALANCE EQUATIONS

state	rate in	=	rate out
0	$\mu_1 P_1$	=	$\lambda_o P_o$
1	$\lambda_o P_o + \mu_2 P_2$	=	$(\lambda_1 + \mu_1) P_1$
2	$\lambda_1 P_1 + \mu_3 P_3$	=	$(\lambda_2 + \mu_2) P_2$
.			
.			
.			
n	$\lambda_{n-1} P_{n-1} + \mu_{n+1} P_{n+1}$	=	$(\lambda_n + \mu_n) P_n$
.			
.			
.			

The balance equations can be solved recursively. The first equation is solved for P_1.

$$P_1 = \frac{\lambda_o}{\mu_1} P_o .$$

Then this result and the second equation can be solved for P_2 in terms of P_o.

$$\mu_2 P_2 = (\lambda_1 + \mu_1) \frac{\lambda_o}{\mu_1} P_o - \lambda_o P_o$$

$$\mu_2 P_2 = \lambda_1 \frac{\lambda_o}{\mu_1} P_o + \frac{\mu_1 \lambda_o}{\mu_1} P_o - \lambda_o P_o$$

$$P_2 = \frac{\lambda_o \lambda_1}{\mu_1 \mu_2} P_o .$$

By continuing this procedure, each P_n can be expressed in terms of P_o .

$$P_3 = \frac{\lambda_o \lambda_1 \lambda_2}{\mu_1 \mu_2 \mu_3} P_o .$$

$$\vdots$$

$$P_n = \frac{\lambda_o \lambda_1 \ldots \lambda_{n-1}}{\mu_1 \mu_2 \ldots \mu_n} P_o .$$

$$\vdots$$

To simplify the notation let

$$C_n = \frac{\lambda_o \lambda_1 \cdots \lambda_{n-1}}{\mu_1 \mu_2 \cdots \mu_n} .$$

Then $P_n = C_n P_o$, for $n \geq 1$.

Since the P_n are probabilities which exhaust all the possibilities,

$$\sum_{n=o}^{\infty} P_n = 1 .$$

Therefore

$$P_o + \sum_{n=1}^{\infty} C_n P_o = 1$$

which implies that

$$P_o = \frac{1}{1 + \sum_{n=1}^{\infty} C_n} .$$

If the λ_n and μ_n are known, then everything else follows from them. The λ_n and μ_n determine the C_n which in turn determine P_o, and P_o and C_n determine the P_n .

Since L is the expected number of customers in the system,

$$L = \sum_{n=o}^{\infty} nP_n \quad .$$

Furthermore since there are S servers in the system, there is no queue unless there are more than S customers in the system. Thus the expected queue length is

$$L_q = \sum_{n=S}^{\infty} (n - S)\, P_n \quad .$$

It is possible to obtain W and W_q from the following results whose proof has been developed elsewhere;*

$$W = \frac{L}{\overline{\lambda}}$$

$$W_q = \frac{L_q}{\overline{\lambda}} \quad .$$

Where $\overline{\lambda}$ is the expected value of the λ_n .

---

* John D. C. Little, "A Proof for the Queueing Formula : L = λW", Operations Research, 9 (3): 383-387, 1961; Shaler Stidham, Jr., "A Last Word on L = λW," Operations Research 22 (2): 417-421, 1974.

All that remains now is to develop expressions for λ_n and μ_n for our queueing model. Suppose that the size of the calling population is M. Then the possible states of the queueing system are 0, 1, 2, ..., M.

In order to obtain an expression for λ_n, assume that a customer's time from leaving the system until returning for the next time has an exponential distribution with parameter λ. If the system is in state n, then n customers are in the system and M-n customers are outside the system. By certain properties of the exponential distribution, the distribution of the remaining time until the next customer arrival is exponential with parameter

$$\lambda_n = (M - n)\lambda \quad , \quad o \leq n \leq M$$

If the rate of service for each individual server is μ, then the rate of service for the system when n servers are busy would be $n\mu$. Since there are S servers, μ_n has the following expression,

$$\mu_n = \begin{cases} n\mu & , \quad 0 \leq n \leq S \\ S\mu & , \quad S \leq n \leq M \end{cases} .$$

An expression for C_n can now be obtained. It is necessary to consider the two cases $n \leq S$ and $n > S$.

First, let $n \leq S$. Then substituting the preceeding expressions for λ_n and μ_n gives

$$C_n = \frac{\lambda_o \ \lambda_1 . . . \lambda_{n-1}}{\mu_1 \ \mu_2 . . . \mu_n}$$

$$= \frac{M\lambda \quad (M-1)\lambda \quad . \; . \; . \quad (M-n+1)\lambda}{\mu \quad (2\,\mu) \quad . \; . \; . \quad (n\mu)}$$

$$= \frac{M!}{(M-n)!\; n!} \left(\frac{\lambda}{\mu}\right)^n \quad , \; n \le S$$

Now consider the case $n > S$.

$$C_n = \frac{M\lambda \quad (M-1)\lambda \; . \; . \; . \; (M-n+1)\lambda}{\mu(2\mu\,) \; . \; . \; . \; (S\mu\,) \quad (S\mu\,)^{n-s}}$$

$$= \frac{M!}{(M-n)!\; S! \quad S^{n-s}} \left(\frac{\lambda}{\mu}\right)^n \quad , \qquad S < n \le M$$

Then

$$P_o = \frac{1}{1 + \sum_{n=1}^{\infty} C_n}$$

$$= \frac{1}{\sum_{n=o}^{s} \frac{M!}{(M-n)!\; n!} \left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s+1}^{M} \frac{M!}{(M-n)!\; S! \; S^{n-s}} \left(\frac{\lambda}{\mu}\right)^n}$$

Once P_o and the C_n have been determined it is simply a matter of substituting in the previously developed equations to obtain P_n, L, and L_q .

$$P_n = C_n \, P_o \quad , \quad 0 < n \le M$$

$$L = \sum_{n=o}^{\infty} n \; P_n$$

$$L_q = \sum_{n=s}^{\infty} (n-s) \; P_n \quad .$$

A simple expression can be obtained for $\bar{\lambda}$, the expected value of the λ_n , by using $\lambda_n = (M-n)\lambda$.

$$\lambda = \sum_{n=o}^{\infty} \lambda_n P_n = \sum_{n=o}^{\infty} (M-n) \lambda P_n = \sum_{n=o}^{\infty} M\lambda P_n - \sum_{n=o}^{\infty} n \lambda P_n$$

$$= M\lambda \sum_{n=o}^{\infty} P_n - \lambda \sum_{n=o}^{\infty} n P_n = M\lambda - \lambda L$$

$$= \lambda (M-L) .$$

Using this value of $\bar{\lambda}$, it is then easy to calculate W and W_q.

$$W = \frac{L}{\bar{\lambda}} .$$

$$W_q = \frac{L_q}{\bar{\lambda}} .$$

3. A DECISION CRITERION FOR THE QUEUEING MODEL

In order to select the optimum float size, some decision criterion is needed. One possible procedure would be to consider all the statistics generated from the preceeding section, and to base the decision on a collective evaluation of those. However that would require the decision maker to juggle several interrelated quantities at the same time and a systematic approach would be more manageable.

As a preliminary, consider the utilization of float aircraft. Float aircraft are intended to replace aircraft which require extended maintenance. Therefore, when an operationally ready float aircraft is needed and none is available, the fleet is deprived of the use of an aircraft. This situation represents an undesirable condition which should be minimized. In order to do that we need some measure of it, and we have one in L_q the expected queue length.

The expected queue length L_q tells on the average how many aircraft are in the queue, ie. in need of a float aircraft when none is available. Suppose, for example, that $L_q = .5$. An expected value of .5 can occur in many different ways. In order to better understand the significance of L_q, note that $L_q = .5$ is equivalent to .5 on an aircraft being in the queue all the time and it is also equivalent to one aircraft being in the queue 50% of the time and no aircraft being in the queue the rest of the time. Thus L_q gives a measure of the need for float aircraft when none is available.

The other side of the coin is represented by the inefficiency of having operationally ready float aircraft available when they are not needed. Again we have an undesirable situation which should be minimized. Let F be the expected number of operationally ready float aircraft which are not being used. To calculate F, recall that the number of servers S is the number of float aircraft. Thus, for example, if there are n customers in the system and $n < S$, then there are $S - n$ available float aircraft not being used. Therefore $F = S\,P_o + (S-1)P_1 + (S-2)\,P_2 + \cdots + 1\,P_{S-1}$. Both L_q and F need to be minimized. However, decreasing one of them increases the other. Therefore a suggested objective function is $Z = L_q + F$ and Z should be minimized.

The objective function in such problems is often taken to be a cost. In our case, Z could be expressed as a cost if L_q were multiplied by the cost of an out of service helicopter and F were multiplied by the cost of an idle helicopter. However, since the out of service and idle helicopters are the same type helicopter, we have taken these costs to be the same (which is the same as not using them). However, for some reason the decision maker may wish to assign different importances to out of service helicopters than to idle helicopters. If so that can be done by assigning costs or simply weighting factors to L_q and F. The objective function would then be $Z = W_1\,L_q + W_2\,F$ where W_1 and W_2 represent the relative importance of L_q and F and need not sum to one.

There are only a finite number of choices for S ($0 < S \leq M$). So to minimize Z, merely calculate Z for the possible choices of S and select the value of S which minimizes Z.

In practice the choices for S may be limited to only 2 or 3 values which further restricts the calculations to minimize Z.

There is another possible criterion for selecting the ideal float size based on the percent of time that float aircraft are available.

Suppose that it is desired that float aircraft be available at least 80% of the time when they are needed. The demands for float aircraft occur at random according to a fixed average rate. The randomness of the demands implies that if float aircraft are available 80% of the time, then 80% of the float demands can be met.

It is easy to calculate the percent of time that float aircraft are available. P_n is the probability that there are exactly n customers in the queueing system, but P_n can also be interpreted as the percent of time that exactly n customers are in the queueing system. There are float aircraft available if $n < s$. Therefore float aircraft are available $P_o + P_1 + \cdots + P_{s-1}$ percent of the time.

The float availability can be calculated for each possible value of S, starting with the smallest value of S, until a value of S is found for which the float availability is greater than 80%. That value of S would be the ideal float size.

Part of the purpose of the examples in section 4 is to illustrate the diversity of the type of problems which can be solved by queueing theory. All of the examples utilize the first described decision criterion. However each of them could just as easily have utilized the second decision criterion.

4. EXAMPLES

In this section some examples are developed to illustrate the use of the queueing model and the decision criterion. Assuming that the fleet size M and the number of servers are known, the only other inputs to the model are λ and μ. Recall that λ is the parameter for the exponential distribution for interarrival times for one single aircraft. This means than λ is the arrival rate for one single aircraft. Suppose that we take our unit of time as one month. Then λ would be the number of times per month that a single aircraft would need a float replacement. Recall that μ is the service rate for one busy server in the system. This means that μ is the number of float replacements that a float aircraft being used continuously could provide in one month.

Example 1: Suppose that there is a fleet of 25 aircraft and that it is desired to determine if it should be supported with a float of 2 or 3 air aircraft. Furthermore, suppose that it is estimated that a single aircraft in the fleet needs a float replacement on the average twice a year and that it is known that when an aircraft is replaced by a float aircraft it takes an average of ½ month to restore it to operationally ready status.

If a single aircraft needs a float replacement twice a year, then the monthly arrival rate is $\frac{2}{12}$. Therefore $\lambda = .167$ arrivals per month. If a float replacement takes ½ month on the average, then a float aircraft could handle 2 replacements a month. Therefore $\mu = 2$ services a month.

The next step is to calculate the objective function $Z = L_q + F$ for the two possible float sizes $S = 2$ and $S = 3$, and select the one

which gives the smaller value of Z. Since there are 25 aircraft in the fleet, M = 25.

Since the calculations involve some unwieldy factorials, they must be done with a simple computer program. The first step is to calculate the C_n from λ and μ

$$C_n = \begin{cases} \dfrac{M!}{(M-n)!\ n!}\left(\dfrac{\lambda}{\mu}\right)^n & , \quad n \leq s \\[2ex] \dfrac{M!}{(M-n)!\ s!\ s^{n-s}}\left(\dfrac{\lambda}{\mu}\right)^n & , \quad s < n \leq M \end{cases}$$

Next the P_o and P_n can be calculated

$$P_o = \frac{1}{1 + \sum_{n=1}^{\infty} C_n}$$

$$P_n = C_n P_o \quad , \quad o < n \leq M$$

The results are shown below for s = 2.

P_o	=	.065	P_9	=	.037
P_1	=	.136	P_{10}	=	.025
P_2	=	.136	P_{11}	=	.016
P_3	=	.131	P_{12}	=	.009
P_4	=	.120	P_{13}	=	.005
P_5	=	.105	P_{14}	=	.002
P_6	=	.088	P_{15}	=	.001
P_7	=	.070	P_n	=	0 , $n \geq 16$
P_8	=	.052			

Recall that P_n is the probability that there are n aircraft in the queueing system. Thus, for example, $P_o = .065$ means that 6.5% of the time all of the float aircraft are idle. Since in this case $s = 2$, a queue exists when $n \geq 3$. This occurs $P_3 + P_4 + P_5 + - - - - = 66.1\%$ of the time.

Then the other quantities follow

$$L = \sum_{n=o}^{\infty} n \; P_n$$

$$L = 4.237$$

If $n > s$, there is a queue. The expected number in the queue is

$$L_q = \sum_{n=s}^{\infty} (n-s)P_n$$

$$L_q = 1(.131) + 2(.120) + 3(.105) + 4(.088) + 5(.070)$$
$$+ 6(.052) + 7(.037) + 8(.025) + 9(.016) + 10(.009)$$
$$+11(.005) +12(.002) +13(.001) = 2.503$$

If $n < s$, there are idle float aircraft. The expected number is

$$F = \sum_{n=o}^{s} (s-n) \; P_n$$

$$F = 2(.065) + 1(.136) = .266$$

Finally $Z = L_q + F = 2.503 + .266 = 2.769$

Using the notation $Z\ (s)$ to indicate that Z is a function of the number of servers gives

$$Z\ (2) = 2.769 \; .$$

Repeating the calculations when s = 3 gives

$$P_o = .120$$
$$P_1 = .250$$
$$P_2 = .250$$
$$P_3 = .160$$
$$P_4 = .098$$
$$P_5 = .057$$
$$P_6 = .032$$
$$P_7 = .017$$
$$P_8 = .008$$
$$P_9 = .004$$
$$P_{10} = .002$$
$$P_{11} = .001$$
$$P_n = 0 \quad , \quad n \geq 12$$

$$L = 2.356$$

$$L_q = 1(.098) + 2(.057) + 3(.032) + 4(.017) + 5(.008)$$
$$+ 6(.004) + 7(.002) + 8(.001) = .465$$

$$F = 3(.120) + 2(.250) + 1(.250) = 1.11$$

$$Z(3) = 1.575$$

Since Z is smaller for s=3, a float size of 3 aircraft is indicated. Of course this result depends upon L_q and F being weighted equally and on the values we have assumed for λ and μ.

Example 2: In example 1, suppose that the value of $\lambda = .167$ is based on the condition that a float aircraft is issued when maintenance of 8 days or longer is required. If the condition is changed to 9 days or

longer, λ will decrease. Indeed, λ is a monotonically decreasing function of X where a float aircraft is issued if maintenance of X or more days is needed. Suppose that the following relation exists

X	8	9	10	11	12
λ	.167	.125	.100	.083	.067

Suppose it is desired to find the number of days X for which the optimum float size decreases from 3 to 2. The procedure is to calculate the objective function Z (2) and Z (3) for the various values of λ in decreasing order until a value of λ is found for which Z (2) < Z (3) . The value of X corresponding to that value of λ is the desired solution.

The following table shows Z (2) and Z (3) for various values of λ.

	s = 2			s = 3		
λ	L_q	F	Z	L_q	F	Z
.167	2.503	.266	2.769	.465	1.11	1.575
.125	.971	.587	1.558	.159	1.54	1.699
.100	.467	.832	1.299	.070	1.814	1.884
.083	.257	1.014	1.271	.035	2.006	2.041

From the table it can be seen that Z (2) < Z (3) for λ = .125 which corresponds to X = 9 days. Thus the desired answer is 9 days. The table also illustrates how L_q and F vary as a function of λ .

Example 3: Use the data from example 2 for λ = .125. In this case Z (2) = 1.558 and Z (3) = 1.699 which indicates that 2 float aircraft should be chosen rather than 3. However that depends on L_q and F being weighted equally. Since the concept of float aircraft entails

having float aircraft in available status to avoid the loss of service of aircraft which require extended maintenance, there is some justification in attaching more importance to the smallness of L_q than to the smallness of F. Suppose that the smallness of L_q is considered twice as important as that of F. The new objective function would than be

$$W = 2 L_q + F .$$

Evaluating W (s) for s = 2 and s = 3 gives

$$W(2) = 2(.971) + .587 = 2.529$$

$$W(3) = 2(.159) +1.54 = 1.858$$

Since W (3) < W (2) , a float size of 3 is now indicated. Note that even though it was desired to minimize the objective function, the more important of the two terms L_q and F is multiplied by the larger weighting factor. Here by changing the weighting factors, the float size increased from 2 to 3 , decreasing L_q and increasing F.

Example 4: In example 1, a float size of 3 is indicated. However suppose that the decision maker believes that by placing more importance on the maintenance of aircraft which have been replaced by float aircraft, the service rate μ can be increased from $\mu = 2$ to $\mu = 2.4$. Taking the other data the same as in example 1, should the float size now be 2 or 3? Calculate Z (2) and Z (3) . Using $Z = L_q + F$,

$$Z(2) = 1.385 + .464 = 1.849$$

$$Z(3) = .237 + 1.389 = 1.626$$

Here the increase in μ is insufficient to decrease the float size; the float size should still be 3.

Example 5: Using $\lambda = .125$ and $\mu = 2$, calculate L_q and F for M = 10, 20, 30, 40, 50 and S = 10% of M for each value of M.

The results are shown in the following table

M	S	Lq	F
10	1	.490	.441
20	2	.420	.848
30	3	.347	1.258
40	4	.287	1.663
50	5	.237	2.073

Note that in each case the float size is exactly 10% of the fleet size. Also note that as the fleet size increases, the expected number in the queue decreases and the expected number of idle servers increases. Thus the larger the fleet, the better a float of a given percentage of the fleet can fulfill the float demands.

In other words, in deciding if a float size of a given percentage of the fleet is correct, the size of the fleet must be taken into account.

5. ALTERNATE MODEL

Implicit in the model which we have used so for is the assumption that aircraft which are in the queue are not receiving maintenance. Depending on the maintenance support available, that might not be true.

If it is desired to allow the aircraft in the queue to receive maintenance while they are in the queue, that can be done by specifying the same number of servers as there are aircraft in the calling population, $S = M$. Then there is always an available server for each aircraft which needs it.

Of course this doesn't represent the situation in the field, and is not intended to. However, it will allow the model to generate statistics about how many servers actually were used and therefore needed. In particular, recall that P_n represents the percent of time that there are n aircraft in the system. In other words P_n is the percent of time that n float aircraft are needed.

It is possible to develop a decision criteria based on the P_n. Suppose that it is desired to have a float aircraft available 80% of the time. As an illustration suppose that the model has produced the following values of P_n.

$P_o = .10$

$P_1 = .15$

$P_2 = .25$

$P_3 = .25$

$P_4 = .10$

$P_5 = .10$

$P_6 = .5$

$P_n = 0 \ , \ n > 6$

If there were 4 float aircraft, there would be float aircraft available when there were o, 1, 2, 3, or 4 aircraft in the system, i.e. $P_o + P_1 + P_2 + P_3 + P_4$ = 85% of the time. If there were only 3 float aircraft, there would be float aircraft available $P_o + P_1 + P_2 + P_3$ = 75% of the time. Therefore the decision criteria would require 4 float aircraft.

In general terms, the P_n should be summed for successive values of n from o up to the smallest value k such that $P_o + P_1 + P_2 + \cdots + P_k > .80$. Then that value of k is the correct number of float aircraft.

6. CONCLUSIONS AND RECOMMENDATIONS

Before giving the conclusions and recommendations, there are some general comments which should be made. In order to illustrate how the model works, it was necessary to have some data.
It seemed that the difficulty of collecting reliable data was not warranted, before the methodology has been accepted for use. Therefore all of the data in the report is hypothetical. However an effort was made to select plausible data.

All statistical models make assumptions about such things as distributions of data. They are more often close approximations to reality than precise descriptions of reality. This is immanent in the nature of statistical analysis and is true for this queueing model. However the degree of closeness of the model to reality is quite good as compared to most modeling efforts.

The model is an attempt to represent the utilization of float aircraft as the author understands the regulations call for them to be used. It is not an attempt to model the situation which exists in the field. Because of the many different influences which exist in the field, distortions occur to the need for and utilization of float aircraft and to the data which describe the needs for and utilization of float aircraft.

The model is a deterministic, expected value model whose results are given in statistical terms. The input and output are average values, and in situations where there is significant variation from the average values, the results of the model could be inaccurate.

CONCLUSIONS:

1) This model provides an excellent methodology for determining the best float size for a fleet of aircraft and to analyze the consequences of floats of different sizes.

2) It requires only two items of input data, the average rate at which a single aircraft needs a float replacement and the average length of time to restore to operationally ready status an aircraft which requires a float replacement.

3) The needed input data should not be hard to collect if indeed it is not available from the present data system.

4) The method allows easy and complete sensitivity analyses.

5) The percentage of a fleet needed for the ideal size of the float depends on the size of the fleet.

6) If there are more than one fleet of aircraft in the same location, it is more efficient (using only considerations of this study) to combine the individual floats to create a large combined float to serve all of the separate fleets than for each individual fleet to have its own float.

RECOMMENDATIONS:

1) This method should be used to evaluate the consequences of floats of various sizes when determining the optimum size of the float.

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